

Advanced Probability Theory 1

MATH 587 – Fall 2023

<http://problab.ca/louigi/courses/2023/math587/>

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Course schedule: Tuesday, Thursday, 14:35-15:55, MH Wong 1040.

Office hours: Wednesday 14:35-15:55, or by appointment.

Course text: My course notes.

Supplemental material is linked from the course website and on MyCourses.

Planned topics

Basic set theory and measure theory, and their interpretations in probability: Dynkin's lemma; uniqueness of extension; Caratheodory's extension theorem; Lebesgue measure. Probability spaces; independence; Borel-Cantelli lemmas, Kolmogorov's zero-one law. Random variables and expectations; inequalities and L^p -spaces; Radon-Nikodym theorem. Sums of independent random variables: types of convergence; laws of large numbers; possibly Levy's equivalence theory. Conditional expectation: definitions; basic properties; martingales; stopping times; martingale convergence; uniform integrability. Further topics should time permit.

Prerequisites.

1. McGill MATH 356.
2. McGill Math 243 or 255. (Two versions of analysis 2.)

On the analysis side, there are a couple of topics I will assume some familiarity with which are covered in Math 255 but not Math 243. In particular, you will be at a disadvantage if you don't already know the basics about metric spaces, completeness and compactness (e.g. the Heine Borel theorem). If you want to brush up on this material, Sam Drury's MATH 354 notes are a good place to look (roughly, chapters 1, 2, and 4.1,4.2,4.5). These notes are available at <http://www.math.mcgill.ca/drury/notes354.pdf>

Instructional methods:

For the first time since 2020, I will be delivering all lectures on the blackboard, and they will not be recorded. Those who are unable to attend some classes will need to use the

other instructional resources (the course textbook and supplemental texts; other online videos) to learn any material from those classes.

I am not 100% happy with this decision, since I recognize that some students learn more easily from videos, and since I do encourage students to stay home when sick for the good of the whole class; however, I tried a hybrid mechanism in Winter 2023; it led to much less overall student engagement with the course, and I believe the overall learning outcomes were harmed.

I have included a number of links to video playlists of probability courses on MyCourses and on my own course website, for those who prefer learning from videos.

Grading Scheme

Mandatory caveat. In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.

(Also, if the resources provided by the university, in the form of teaching assistants/graders, are inadequate, the grading scheme may be modified.)

Your grade will be calculated as the maximum of the following three formulas:

Formula 1:

- Marked homework assignments: 20%
- Midterm: 30%
- Final: 50%

Formula 2:

- Marked homework assignments: 20%
- Midterm: 20%
- Final: 60%

Formula 3:

- Marked homework assignments: 25%
- Midterm: 0%
- Final: 75%

The midterm will likely take place on the evening of October 18. Students who are unable to or who choose not to write the midterm for any reason will be evaluated according to Formula 3. There will be no deferred or supplemental midterm exam.

Assignments are to be submitted via Crowdmark. Typesetting your assignment gives a 10% bonus to that assignment's grade. A LaTeX assignment solution template is available from the course website.

According to the [university student assessment policy](#), "penalties for late submitted as-

signments must be stated in the course outline”. The course policy is that all late submissions receive a grade of 0.

Assignments will be judged on mathematical correctness and completeness and also on clarity of exposition. Clarity of explanation will be judged according to the standards of the book [Mathematical writing for undergraduate students](#). Writing that is similar to that in the “bad” examples from that book, and writing of a style that the book specifically suggests avoiding, may cause loss of marks.

The midterm will be held in mid-October, likely from 6-8 PM on October 18. Students who are unable to or who choose not to write the midterm for any reason will be evaluated according to There will be no supplemental midterm; any student missing the midterm will be graded according to option (C).

Students may work in groups and may submit assignments in groups. If an assignment is co-written by a group, please *only submit it once*; the ID numbers of all students working in the group should then appear on the write-up. However, if you work in a group but do your own write-up, you should submit that individually.

McGill’s “student rights and responsibilities” web page has this to say:

Peer learning should be encouraged, since it helps students learn to teach. Instructors should explain effective peer teaching strategies such as working in pairs, sharing comments on work, and brainstorming solutions to problems in groups. Sharing completed work is not an acceptable peer learning technique. If a student has copied the answers of another student, the incident must be documented and the material sent to the course instructor, who will contact the appropriate disciplinary officer.

–From <https://www.mcgill.ca/students/srr/honest/students/test/ugrad/sharing-your-work>

Language

In accord with McGill University’s Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded. Conformément à la Charte des droits de l’étudiant de l’Université McGill, chaque étudiant a le droit de soumettre en français ou en anglais tout travail écrit devant être noté (sauf dans le cas des cours dont l’un des objets est la maîtrise d’une langue).

Academic Integrity

McGill requires that the following statement be included in all course outlines. “McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see www.mcgill.ca/students/srr/honest/)

for more information).

L'université McGill attache une haute importance à l'honnêteté académique. Il incombe par conséquent à tous les étudiants de comprendre ce que l'on entend par tricherie, plagiat et autres infractions académiques, ainsi que les conséquences que peuvent avoir de telles actions, selon le Code de conduite de l'étudiant et des procédures disciplinaires (pour de plus amples renseignements, veuillez consulter le site www.mcgill.ca/students/srr/honest/)

However: *I do not intend* to spend my time policing plagiarism/“cheating” in this class. I want to spend my time helping you learn, not being an academic integrity cop. Any other members of the instruction team (TAs, graders) will also not have searching for or reporting cheating instances as part of their job description; they will also spend their energy on tasks that more directly help you to learn the course material.

The following document informs my approach to questions of “academic integrity”:
<https://www.justmathematicscollective.net/cheating.pdf>

Finally, all the work you hand in for the course must be your own (or your group's, in the case of group work). If you hand in work that is not your own/your group's, you will not receive credit for that work.

Class environment

I wish for everyone in my class to feel welcome and welcomed by their instructors and by their peers. This means:

- If you notice classmates struggling, it is great to offer help.
- If you are struggling, it is great to ask for help (me or the TA or your classmates).
- When working with other class members, try to be positive rather than critical. For example, try to avoid saying things like “this is trivial” – all it can accomplish is to make others feel bad. Maybe try explaining the concept instead (if your co-workers want you to!).
- There are multiple resources available for you to
- Finally: don't harass or make other unwanted advances to your classmates. (This of course applies more generally!)

On the topic of harassment, McGill also recently (August 21) emailed instructors the following text:

HARASSMENT: There have recently been a small number of unfortunate incidents in which Teaching Assistants have suffered verbal abuse from undergraduate students they are instructing. The University takes this very seriously. If you become aware of this occurring in your class, please let me know immediately so that appropriate action can be taken. You might also want to include the following text adapted from TLS in your course syllabus:

“The University is committed to maintaining teaching and learning spaces

that are respectful and inclusive for all. To this end, offensive, violent, or harmful language arising in course contexts may be cause for disciplinary action under the Article 10 of the Code of Student Conduct and Disciplinary Procedures and Section 2.7 of the Policy on Harassment, Sexual Harassment, and Discrimination Prohibited by Law.”

To that I would add that *you* also deserve not to be harassed: by me or any member of the teaching/instructional staff, or by your peers. The Office for Mediation and Reporting (<https://www.mcgill.ca/omr/harassment-discrimination-0>) can advise you on McGill’s processes related to harassment and discrimination. You are also welcome to talk to me about any class-related situations that arise during term, if you feel comfortable doing so.