MATH 589 - ADVANCED PROBABILITY THEORY 2

Winter, 2020

http://problab.ca/louigi/courses/2019/589/

Professor: Louigi Addario-Berry, louigi@problab.ca, Burnside 1219
Course schedule: Tuesday, Thursday, 13:00-14:30, Burnside 1205.
Office hours: Tuesday, Thursday, 14:30-15:15, Burnside 1219 or by appointment.

TEXTBOOK

Richard Durrett's "Probability, Theory and Examples", available on the author's website:
https://services.math.duke.edu/ rtd/PTE/PTEfive.pdf
I will not follow this text particularly closely, and hope to also post my own notes, but it is a useful reference.

PREREQUISITES

MATH 587 or equivalent. I will teach this course as a continuation of the MATH 587 course I taught in Fall 2019. You should be familiar with the material covered in the lecture notes for that course, up to and including Doob's upcrossing inequality and the nonnegative martingale convergence theorem.

PLANNED TOPICS

Martingales (2 weeks)
More martingale convergence theorems. Examples: Dirichlet problem for Markov chains; martingale proof of Radon-Nikodym; the Kesten-Stigum theorem for branching processes.

Transforms (3-4 weeks)
Moment generating functions; characteristic functions. Elementary properties, Inversion formula, Uniqueness, convolution and continuity theorems. Convergence of moments. Central limit theorem.

Weak convergence (3-4 weeks)
Portmanteau theorem, Tightness, Prohorov's theorem, Polish spaces, Skorokhod's representation theorem.

Stochastic processes (2-3 weeks)
General theory. Kolmogorov Extension theorem, Kolmogorov continuity theorem. Regular probability spaces and conditional distributions, probability kernels.

Exchangeability (0-2 weeks)
De Finetti's theorem, The Aldous-Hoover theorem.

Other topics if time permits.

GRADING SCHEME

Your grade will be calculated according to the more favourable of the following two options:
(A) Assignments 20%, midterm 20%, final 60%
(B) Assignments 25%, final 75%

Submitting an assignment written in LaTeX gives a 10% bonus to that assignment's grade. A LaTeX assignment solution template is available from the course website.
The penalty for late assignments is 10% per day.
Five total days of non-penalized lateness are allocated to each student. Additionally, assignments written in LaTeX and submitted by email may be 24 hours late without penalty; this won't count against the aforementioned five days.

Assignments will be judged on mathematical correctness and completeness and also on clarity of exposition. Clarity of explanation will be judged according to the standards of the book Mathematical writing for undergraduate students. Writing that is similar to that in the "bad" examples from that book, and writing of a style that the book specifically suggests avoiding, may cause loss of marks.

Students may work in groups and must indicate who they worked with on their submitted assignment. I encourage you to read over one-another's assignments and give feedback, but you should each write up your assignment solutions on your own. McGill's "student rights and responsibilities" web page has this to say:

> Peer learning should be encouraged, since it helps students learn to teach. Instructors should explain effective peer teaching strategies such as working in pairs, sharing comments on work, and brainstorming solutions to problems in groups. Sharing completed work is not an acceptable peer learning technique. If a student has copied the answers of another student, the incident must be documented and the material sent to the course instructor, who will contact the appropriate disciplinary officer.
> –From https://www.mcgill.ca/students/srr/honest/students/test/ugrad/sharing-your-work

Missing an assignment without a medical note will result in a grade of zero for the assignment.

The midterm will be held in class on October 22. There will be no supplemental midterm; any student missing the midterm will be graded according to option (B).

LANGUAGE

In accord with McGill University's Charter of Students Rights, students in this course have the right to submit in English or in French any written work that is to be graded.
Conformément à la Charte des droits de l'étudiant de l'Université McGill, chaque étudiant a le droit de soumettre en français ou en anglais tout travail écrit devant être noté (sauf dans le cas des cours dont l'un des objets est la maîtrise d'une langue).

ACADEMIC INTEGRITY

McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see www.mcgill.ca/students/srr/honest/ for more information).
L'université McGill attache une haute importance à l'honnêteté académique. Il incombe par conséquent à tous les étudiants de comprendre ce que l'on entend par tricherie, plagiat et autres infractions académiques, ainsi que les conséquences que peuvent avoir de telles actions, selon le Code de conduite de l'étudiant et des procédures disciplinaires (pour de plus amples renseignements, veuillez consulter le site www.mcgill.ca/students/srr/honest/