FALL 2016. MATH 598 - BROWNIAN MOTION

Professor: Louigi Addario-Berry, louigi.addario@mcgill.ca, Burnside 1219
Course schedule: Monday, Wednesday, 10:05-11:25, Burnside Hall 1205.
Office hours: by appointment, or drop by.

History.
"Extremely minute particles of solid matter, whether from organic or inorganic substances, when suspended in pure water, or in some other aqueous fluids, exhibit motions for which I am unable to account, and which from their irregularity and seeming independence resemble in a remarkable degree the less rapid motions of some of the simplest animalcules of infusions." –Robert Brown, 1829.

The theory of Brownian motion is one of the great interdisciplinary success stories of mathematics. After the initial observations by Brown (a biologist) and important, independent contributions by Thiele (statistics), Bachelier (mathematical finance), Einstein and Smoluchowski (physicists) in the period 1880-1910, a rigorous construction was given by Norbert Wiener (mathematician) in 1923. Today, the theory of Brownian motion plays an important role in all these fields, and in many more.

Outline. This course will rigorously introduce and describe the fundamental properties of Brownian motion and related stochastic processes, in particular:
⋆ Construction of Brownian motion, basic properties of Brownian sample paths.
⋆ Brownian motion as a Markov process; Brownian motion as a martingale.
⋆ Continuity properties, dimensional doubling
⋆ Donsker's invariance principle, arcsine laws
⋆ The law of the iterated logarithm
⋆ Recurrence and transience, occupation measures and Green's functions
⋆ Brownian local time
⋆ Stochastic integrals with respect to Brownian motion; Tanaka's formula; Feynman-Kac formulae

Some of the following topics will also be addressed, time permitting.
⋆ Hausdorff dimensions of (subsets of) Brownian motion sample paths
⋆ Polar sets, intersections and self-intersections of Brownian motion:
⋆ Fast times and slow times.
⋆ The Brownian continuum random tree
⋆ Introduction to SLE
⋆ Introduction to the theory of continuous martingales.
⋆ Introduction to Lévy processes
⋆ Itô's excursion theory for Brownian motion.
⋆ Gaussian processes, the Gaussian free field.

Textbook. *Peter Mörters and Yuval Peres*, Brownian Motion. Additional handouts may be provided during the course.

Grading Scheme. The grading scheme is as follows: Class participation, 25%; Assignments, 75%.

Prerequisites. Math 587 or permission of the instructor.

Additional mandatory information. 1. In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

2. McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see www.mcgill.ca/students/srr/honest/) for more information).

3. In the event of extraordinary circumstances beyond the Universitys control, the content and/or evaluation scheme in this course is subject to change.